Dispersion risk

Helen Hizhniakova and **Tatiana Lozovaia*** look at how to extend modern portfolio theory to make money from trading equity options

Every trader, market maker or financial analyst knows what risk is and knows the methods to estimate it. There are various theories on how to estimate risk in the world of finance. We do not endeavour in this article to evaluate some of the more sophisticated theories of risk estimation. However, a brief overview of what is probably the best known theory of portfolio risk, Value at Risk (VaR), is warranted. VaR has become so prevalent that it is almost impossible to find professionals in the financial markets that are not at least remotely familiar with the measure.

What is commonly referred to as VaR, is the risk expressed in dollar terms showing what amount of money a portfolio can lose during a defined interval with a given probability. The terms that are most

See text for formula references

1.

$$\sigma_p^2 = \sum_{i=1}^{N} W_i^{\,2} * \sigma_i^{\,2} + 2\sum_{i=1}^{N}\sum_{j>1} W_i * W_j * \sigma_i * \sigma_j * corr_{ij}$$

2.

$$\text{Implied Index Correlation} = \frac{\sigma_I^{\,2} - \sum_{i=1}^{N} W_i^{\,2}\sigma_i^{\,2}}{2\sum_{i=1}^{N}\sum_{j>1} W_i W_j \sigma_i \sigma_j}$$

3.

$$IIC > 1 \Rightarrow \sigma_I^2 > \sum_{i=1}^{N} W_i^2 \sigma_i^{\,2} + 2\sum_{i=1}^{N}\sum_{j>1} W_i W_j \sigma_i \sigma_j > \sum_{i=1}^{N} W_i^2 \sigma_i^{\,2} + 2\sum_{i=1}^{N}\sum_{j>1} W_i W_j \sigma_i \sigma_j \; corr_{ij} = \sigma_p^2$$

4.

$$\text{WtdCompIV} = \sum_{i=1}^{N} W_i * IV_i$$

5.

$$\sigma_I^2 = \sum_{i=1}^{N} W_i^{\,2}\sigma_i^{\,2} + 2 * IIC * \sum_{i=1}^{N}\sum_{j>1} W_i W_j \sigma_i \sigma_j$$

6.

$$\sigma_I^2 = \Big(\sum_{i=1}^{N} W_i^{\,2}\sigma_i\Big)^2 + 2 * (IIC-1) * \sum_{i=1}^{N}\sum_{j>1} W_i W_j \sigma_i \sigma_j$$

7.

$$\sigma_p = \sqrt{\sum_{i=1}^{N} W_i^{\,2} * \sigma_i^2 + 2\sum_{i=1}^{N}\sum_{j>1} W_i * W_j * \sigma_i * \sigma_j * corr_{ij}}$$

8.

$$\text{CorrWtdCom pIV}_p = \sqrt{\sum_{i=1}^{N} W_i^{\,2} * IVIndex_i^2 + 2\sum_{i=1}^{N}\sum_{j>1} W_i * W_j * IVIndex_i * IVIndex_j * corr\,(IV_i\,, IV_j)}$$

9.

$$CorrWtdCom\ pIV^2 = \sum_{i=1}^{N} W_i^{\,2} IV_i^{\,2} + 2\sum_{i=1}^{N}\sum_{j>1} W_i W_j IV_i IV_j\, corr_{ij} \le \sum_{i=1}^{N} W_i^{\,2} IV_i^{\,2} + 2\sum_{i=1}^{N}\sum_{j>1} W_i W_j IV_i IV_j = WtdCompIV^2$$

10.

$$\text{CorrWtdCompHV}_p = \sqrt{\sum_{i=1}^{N} W_i^{\,2} HV_i^2 + 2\sum_{i=1}^{N}\sum_{j>1} W_i W_j HV_i HV_j \;\; corr(price_i\,, price_j)}$$

commonly mentioned along with VaR (and are similar in meaning to VaR) are dispersion, variance and volatility. So, VaR is, in essence, the volatility of the portfolio expressed in dollar terms. Beginning with the basics of VaR analysis, we can demonstrate how this concept can be applied not only to estimate the risks of a portfolio of assets but also how this theory can be applied to trading the market itself.

In general, we could consider the markets as one large portfolio of various assets with each having their respective weights in the global marketplace. Furthermore, we can estimate the global market risk, draw comparisons with historical data and perform many types of empirical studies. The only significant practical difficulties that one would encounter would be the large quantity of historical information required and the substantial attendant technical resources to manage it.

We can more efficiently explore the same concepts by opting to focus on a few subsets of the global markets. Let's consider a few major indices within the markets, which were introduced specifically to provide an assessment of the general market conditions, and let's venture to apply dispersion analytics to these indices.

An index measures the price performance of a portfolio of selected stocks. It allows us to consider an index as a portfolio of stock components. From that point of view, index risk can be evaluated as the risk of the portfolio of stock constituents.

It is well known that the portfolio risk is a weighted sum of the covariation of all stocks in the portfolio. Thus, it can be calculated by formula 1 (see box on page 57 for all equations) that hereinafter will be referred to as the main formula.

Here w_i is a weight of an asset in the portfolio or a component weight in the index, σ^2_i is the dispersion of a stock component[i]. With this formula the portfolio or index risk can be calculated for different terms, since we can use volatilities and correlations calculated on any desired historical time interval for price data and different forecast times for implied data. Also, by using this formula we can calculate a value that expresses the correlation level between the IVs of the stocks and the index IV observed on the market.[ii]

Fundamental index parameters

Let's discuss the types of values that can be employed in the dispersion strategy. These values enable traders to determine whether current conditions are suitable for a dispersion trade. We will distinguish amongst these three kinds of values: realised, implied, and theoretical:

- ❑ Realised values can be calculated on the basis of historical market data, ie, prices observed in the market in the past. For example, values of historical volatility and correlations between stock prices are realised values
- ❑ Implied values are values implied by the option prices observed on the current day in the market. For example, IV of stock or index is volatility implied by stock/index option prices, implied index correlation (IIC) is an internal correlation implied by the market
- ❑ Theoretical values are values calculated on the basis of a particular theory, so they depend on the theory chosen to calculate them. Index volatilities calculated on the basis of the portfolio risk formula are theoretical, and can differ from realised or IVs.

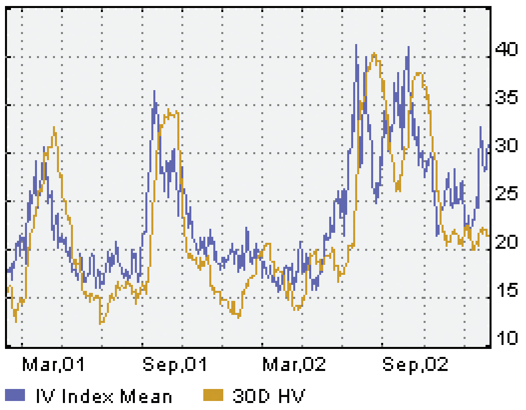
A comparison of theoretical values with realised ones allows traders to determine what market behaviour is best applied to the actual trading environment. By studying and analysing the historical relationship between these two types of values one can make an informed decision about related forecasts.

By comparing implied and historical volatilities, theoretical and realised, or theoretical and implied values of the index risk, we can attempt to ascertain the best time to employ the dispersion strategy or to choose to continue to monitor the markets.

The dispersion strategy typically consists of short selling options on a stock index while simultaneously buying options on the component stocks, the reverse dispersion strategy consists of buying options on a stock index and selling options on the component stocks.

Over the last two years, index options were priced quite high (as shown in the charts), while historical volatility was lower. As a result, it was generally profitable to sell rich index options. However, there were also short periods, when the reverse scenario occurred, thus buying index options in those periods would have generally resulted in profits.

Chart 1. IVIndex and HV for DJX



As can be seen from chart 1, the historical volatility of DJX was greater than the IV at times and reached a local maximum in the last week of September 2001. The explanation for this lies in the tragic events of September 11. A similar jump in volatility, which was driven by very different factors, can be observed in the September 2002 segment. This particular time was good for buying cheap index options and, thus, engaging in the reverse dispersion strategy.

Historical volatility

Historical volatility (HV) is calculated on the basis of stock price changes observed over a given time period. Prices are observed at fixed intervals of time (named terms). For example, every day, every week and every month. HV is calculated as the standard deviation of a stock's returns for the last N days. Return is defined as the natural logarithm of close-to-close price observations.

IV Index

The Implied Volatility index, or IVIndex, is the main parameter used for implied data in dispersion strategy analysis. IV is volatility that is implicit in the option prices observed within the markets. IV can be used to monitor the market's opinion about the volatility of a particular stock or index. In addition, IV values may be used to estimate the price of one option from the price of another option.

As IVs are different for different options, it is useful to have a composite volatility for a stock/index by taking suitable weighted individual volatilities. Such a composite volatility, calculated on the basis of 16 vega weighted ATM options and normalised to a fixed maturity, is called the IVIndex. Hereinafter, when we refer to the IV of a stock or index we mean the IVIndex.

Chart 2 shows the time history of the IVIndex for the OEX, DJX, SPX, SOX, NDX and OSX indices. As one can observe from the chart:

- ❑ IVs of indices generally move synchronously
- ❑ The IVIndex of global equity indices such as SPX, OEX and DJX are almost coincident, since such indices reflect the state of economics in general, and so their performances are affected by the similar factors
- ❑ The IVs of global equity indices are lower than those of sector indices (for example, see SOX and OSX). This is because changes in one sector significantly impact indices from this sector but may also slightly influence the global market indices. So the risk for the index that consists of equities from within the same industry is higher
- ❑ As can be seen, the Nasdaq-100 (violet line) has higher IV than other major market indices. This is because this index includes companies listed on the Nasdaq Stock Market only, while DJX, OEX and SPX cover a broader group of stocks listed on different exchanges.

Major market indices have, on the whole, lower IV in comparison with sector indices. That is not to suggest that some indices are better and others worse for the dispersion strategy. The important role in the strategy concerns not the absolute value of IV but the historical relationship between implied and realised index volatility.

Example

The IVIndex and HV of the DJX index can be seen in chart 1. The most recent IV of the index (31%) is not far from the recently registered maximum value (41%) and it reaches its local maxim and higher than realised (historical) volatility. It suggests that it is a good time to sell options on the DJX Index in the dispersion strategy because it means selling relatively rich options[iii].

Implied index correlation

IIC defines correlation levels between the actual IV of the index and the IV of its stock components. In other words, it is a component-averaged correlation between IVs calculated from the formula of the portfolio risk (see formula 2), where σ_I and σ_i are actual index and stock components' IVs and w_i is a weight of a component in the index.

Chart 2. Time history of IV for OEX, DJX, SPX, SOX, NDX and OSX.

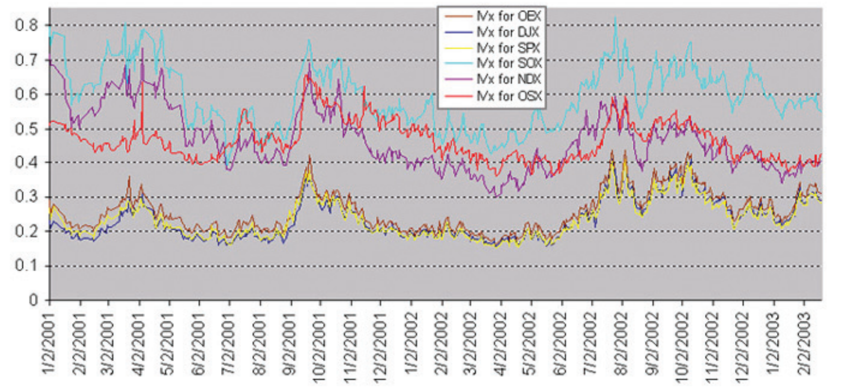


Chart 3. IIC of OSX

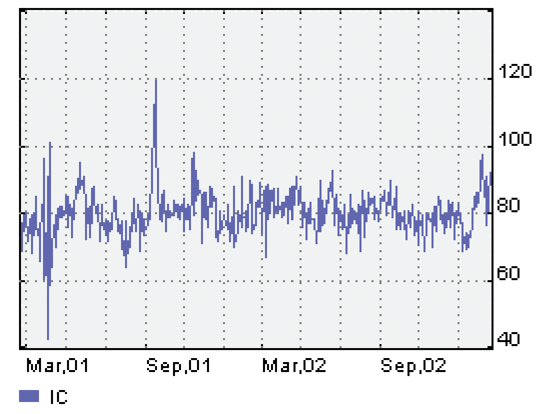
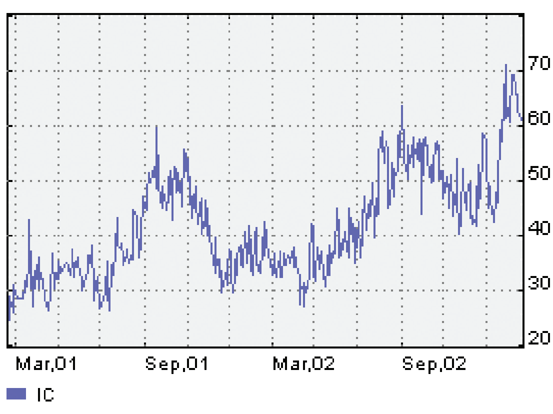


Chart 4. IIC of OEX



The greater IIC, the stronger correlation between the index IV and that of its constituent stocks and, therefore, the more suitable the market conditions for deploying a dispersion strategy. However, this is not an absolute measure and should, therefore, be examined in the light of its historical (realised) performance.

Realised index correlation can be calculated from the same formula by using the historical volatilities of stocks and the index instead of IVs.

Empirically, sector indices have usually exhibited higher implied and realised correlations than major market indices. The 30 day IIC of sector indices SOX and OSX are 75.75% and 93.9% correspondingly, while the implied correlation of broader market indices such as the NDX, OEX and DJX are lower (60.87%, 66.4% and 68.5% correspondingly). The reason for this phenomena is that sector indices consist of equities from within the same industry, thus they are much more co-dependant on the same conditions rather than equities from different industries.

Looking at the time history of IIC for global market indices (DJX, OEX,SPX etc) one can observe that the IIC has a tendency to increase (there are some jumps and drops but on whole it rises), while the average IIC of sector indices (OSX, SOX etc) remains on the same high level (see charts 3 and 4).

The IIC is calculated on the basis of the IVs of the stock components and the index options that are implicit in the option prices observed in the market. So, the IIC can be greater than one, or 100%, since the individual stock options and index options markets are, in reality, separate markets. IIC greater than one means that the actual IV of the index substantially exceeds the theoretical volatility, ie, theoretically, index options

Chart 5. IIC for DJX

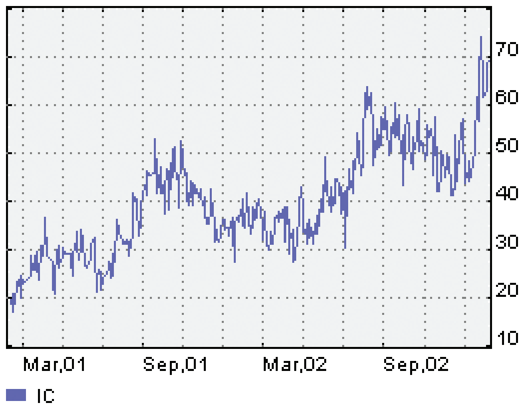
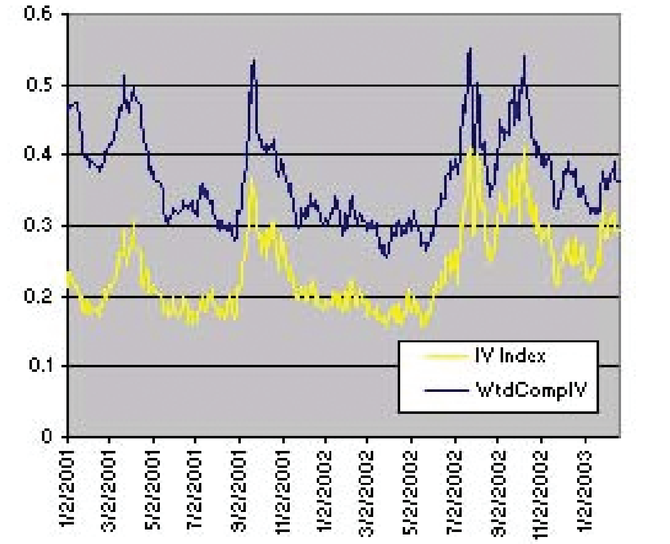


Chart 6. Theoretical and IVs of DJX



are overpriced (see formula 3). But as a rule, for American indices IIC is less than 100%.

In this formula σ_I is the actual IV of index calculated from the market conditions, and σ_p is the theoretical IV of the index calculated from the formula of the portfolio risk.

Example

The greater the IIC, the greater dependence between volatilities of index and its components. Therefore, the higher the average correlation between index volatility and the volatilities of constituent stocks, the better the timing of a dispersion strategy. Let's look at chart 5, which shows the 30 day IIC for the DJX index. Last year, the correlation was positive and the current value of 70% is close to the maximum. It suggests an opportune time for a dispersion strategy.

Theoretical volatility values

There are several ways to calculate the risk of the index. Theoretical index volatility can be calculated from the formula of portfolio dispersion, or as a weighted sum of the components' volatilities. The calculation of theoretical volatilities is not difficult but it is a very laborious task, since it requires the processing of a substantial quantity of historical data.

The simplest method to calculate index volatility is to consider it as a weighted sum of the volatilities of its components. This sum is called the weighted components' IV (WtdCompIV), or the weighted volatility of the index (see formula 4).

Weighted volatility – The weighted volatility of the index, calculated this way, expresses the overall IV of the index components but ignores the correlation between the component stocks. The ratio of the components' IV (WtdCompIV %) to actual implied index volatility will be referred to as a first volatility level coefficient.

Let's clear up the connection between the weighted component IV and IIC described above. As it follows from the formula of IIC (see formula 5), where σ_I and σ_i are actual values of the index and stock components IVs, IC is implied index correlation. It is not difficult to re-arrange this expression (see formula 6).

So, if IIC is one, the actual IV of the index is exactly the weighted sum of the individual stock constituent IVs (see formula 6). As IC is, generally, less than one the first volatility level coefficient is greater than 100%, ie the theoretical volatility of the index calculated without taking correlations between index components is greater than actual IV (see chart 6). But, since IIC can be greater then 100%, (for example, this happens for European indices), the first volatility level coefficient can be less than one.

Theoretical implied volatility – The main factor affecting the first volatility level coefficient is the changing correlation between stock and index volatility. Chart 6 shows the components' IV and the actual IV for DJX. As can be seen on the chart, on the whole, components' IV and actual IV move synchronously. But, lately, the weighted components' IV verges towards the actual IV (see chart 7). This can be explained by the growing IIC (see chart 8).

Chart 7 displays the first volatility level coefficient for the DJX index. The coefficient falls over the last two years. The current value of the coefficient is

Chart 7. First volatility level coefficient for DJX

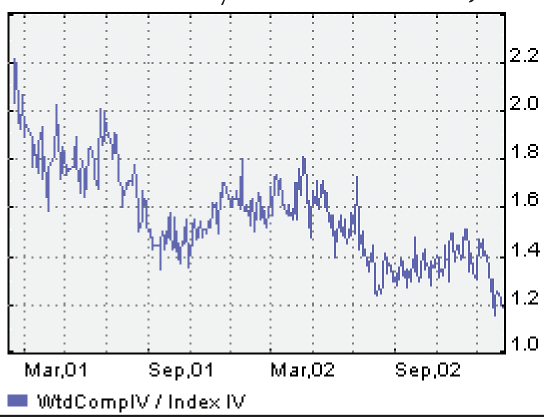


Chart 8. IIC for DJX

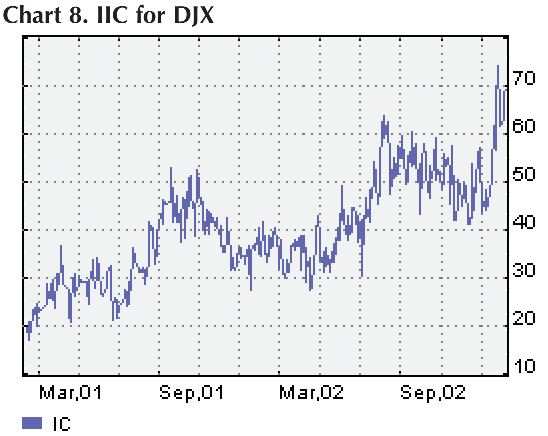


Chart 9. Second volatility level coefficient for SOX

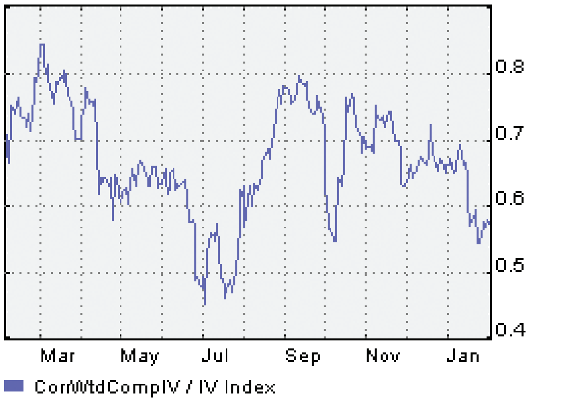
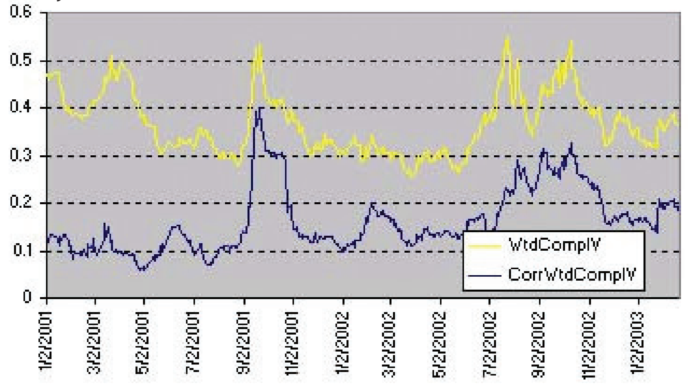


Chart 10. Time history of WtdCompIV and CorrWtdCompIV for DJX



1.2. It is close to its lowest value in two years. Thus, it is a convenient time for the dispersion strategy because it means that the correlation level between the IVs of stocks and the index is high.

Let's consider the index as a portfolio of component stocks with the corresponding weights. Thus, by using the main formula we can calculate the risk of the index as the risk of the portfolio (see formula 7).

We can calculate the theoretical value of index volatility on the basis of the IVIndex of each component and the correlations between the components' IVIndex values (see formula 8). This volatility will now be referred to as the theoretical correlated IV of the index.

Here w_i is a component weight in the index, $IVIndex_i$ is the IVIndex of a component stock and $corr(IV_i, IV_j)$ is the correlation between the IVIndices of two stocks.

The ratio of the theoretical correlated IV of the index to the actual IV is calculated to estimate the difference between the theoretical and real prices of index options. This relationship is called a second volatility level coefficient.

If the second volatility level coefficient is less than one and relatively low, it means that, theoretically, index options are overpriced. Thus it is profitable to sell index options. If the second volatility level coefficient is greater than one and relatively high, it means that, theoretically, index options are conservatively priced and represent a value. Thus it is profitable to buy them.

In practical terms, since the second volatility level coefficient is based on correlations of IVs, it is a better measure for a vega trade. Longer term trades have lower gamma but higher vega and so the predominant risk factor is volatility. The second volatility level coefficient tends to perform better for longer term trades where vega is the prevalent risk factor while gamma and theta are minor risk factors. Comparing second volatility level coefficients for different terms allows the trader to select the best term for dispersion trades.

Example

Chart 9 shows the second volatility level coefficient of SOX for a 30 day term. The current value is 0.57. So, historically, theoretical correlated IV is lower than actual IV and this has been observed for all indices over last few years. Nevertheless, drops and jumps over histories can show times where, theoretically, options were more or less overpriced. As a result, selling index options is profitable in these periods.

It is simple to prove that the theoretical correlated IV of an index, which is calculated on a correlation adjusted basis, cannot be greater than weighted IV, which ignores correlations (see formula 9).

Thus, the second volatility level coefficient is always less or equal to the first coefficient. As can be seen in Chart 10, taking into account the correlation between stocks essentially reduces overall theoretical index volatility.

Theoretical historical volatility – The theoretical HV of the index can be calculated from the main formula on the basis of the HVs of each component and the correlations between stock prices (see formula 10).

In this formula, HV_i is the HV of constituent stocks calculated on the basis of recent 10, 20, 30, 60, 90, 120, 150 and 180 days, $corr_{ij}$ is the correlation between the stock prices for the corresponding term.

The ratio of the theoretical historical volatility of the index to actual is shown in Chart 11. This ratio (called the third volatility level coefficient) indicates how much theoretical HV differs from actual volatility.

If the third coefficient is less than one and low, it means that the theoretical performance of the index is less than the actual volatility and the trader can gain profit by selling index options. As a rule, for American indices the third volatility level coefficient is less than one. The higher the value of the third volatility level coefficient, the better the time for buying index options.

Shorter term options have much less vega risk but more gamma risk, ie risk to the price moves of the underlying. On a practical level, since the third volatility level coefficient is based on stock prices and the correlations between them, it is a better measure for a gamma trade, ie a trade in which you are trying to capture the relative value based solely on gamma. Thus, this measure tends to be better suited for short term portfolios where gamma is the dominant factor.

Chart 11. Third volatility level coefficient for DJX

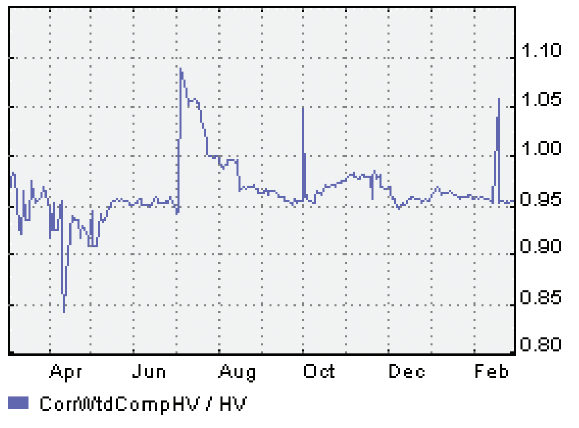
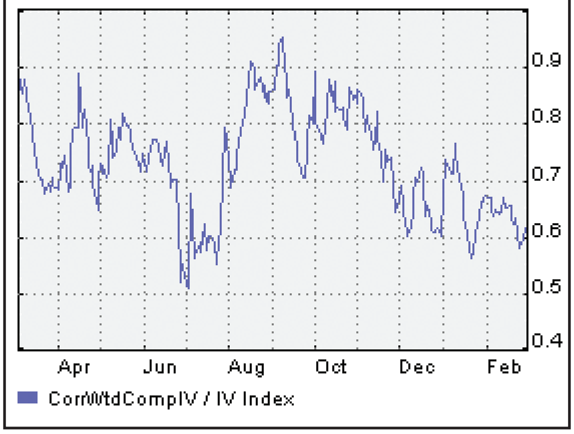


Chart 12. Second volatility level coefficient for DJX



Example

Chart 11 (on page 61) shows the third volatility level coefficient for DJX. The Current value is relatively high and less than one, thus it is not a good time to sell short term index options, since the theoretical HV is not far from actual one. However, as the current value of the second volatility coefficient is relatively low and less than one, selling long term index options can be profitable, because, theoretically, they are substantially overpriced.

Chart 13 shows the theoretical IV (CorrWtdCompIV) and the theoretical HV (CorrWtdCompHV) for DJX. As a rule, CorrWtdCompHV (calculated on the basis of price correlations and HV) is higher than CorrWtdCompIV (calculated on the basis of IVs and the correlations between them). It arises from the higher level of correlations between the stocks' prices in comparison with the correlations of the stocks' IVs.

Dispersion strategy

Volatility dispersion strategy is considered to be one of the best working strategies in sophisticated analytics. It can be explained by the fact that, historically, index volatility has traded rich, while individual stock volatility has been fairly priced. Thus, the dispersion strategy allows traders to profit from price differences using index options and offsetting options on individual stocks.

The dispersion strategy typically consists of short selling options on a stock index while simultaneously buying options on the component stocks, ie short correlation and long dispersion. The reverse dispersion strategy consists of buying options on a stock index and selling options on the component stocks.

The success of the volatility dispersion strategy lies in determining whether the time is right to do a dispersion trade and selecting the best possible stocks for the offsetting dispersion basket.

Effective timing

When selecting the best time to engage in the dispersion strategy, the trader should pay attention to the following parameters:

❑ The IVIndex of the index – if the relationship of the actual IV of the index to the HV is greater than one and relatively high, it is a good time to sell index options, as it means selling expensive options on the stock index. If the relationship of the actual IV of the index to the HV is less than one and relatively low, it is a good time to buy index options, as it means buying relatively cheap options

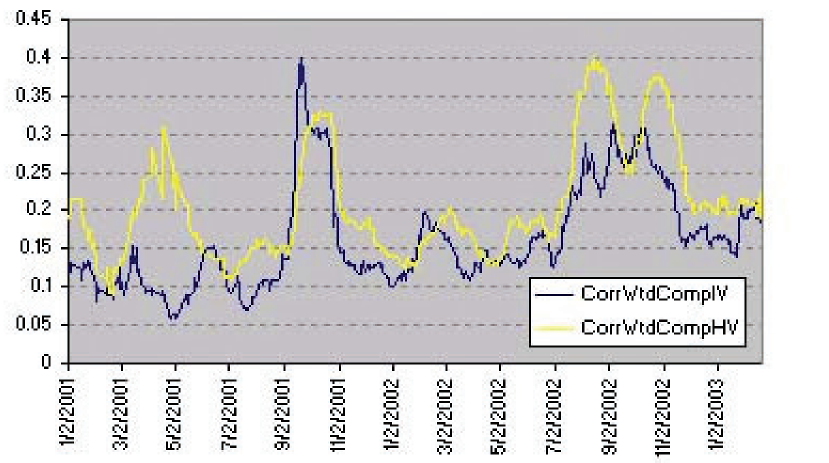
❑ IIC – this should not be too far from the maximum registered value, since the dispersion strategy works better if the IV of the index is highly correlated with the IVs of its stock components

❑ WtdCompIV/IVIndex – first volatility level coefficient. The low value corresponds to high IIC. So, the lower the value of the first volatility level coefficient, the better the time to engage in a dispersion strategy

❑ CorrWtdCompIV/IVIndex – the second volatility level coefficient is a better measure for the longer term trade. If the second volatility level coefficient is less than one and relatively low, it means that the theoretical performance of the index is less than implied by the market and a trader can gain profit by selling index options. Otherwise, if the second coefficient is greater than one and relatively high, it is a better time to buy index options

❑ CorrWtdCompHV/HV – the third volatility level coefficient is a better measure for shorter term trade. If the third volatility level coefficient is less than one and relatively low, it means that theoretical performance of the index is less than implied by market and a trader can gain profit selling index options. Otherwise, if the third coefficient is greater than one and relatively high, it is a better time for buying index options.

Chart 13. CorrWtdCopmIV and CorrWtdCompHV for DJX



Component stocks

Assuming that the timing is propitious for a dispersion trade, the next step is to select the best component stocks to sell (or buy in the case of reverse dispersion strategy). ❑

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[i] The formula can be used in several ways. First, all the data in this equation are provided by observations of the market. But if we were to input actual data on the left and right side, we would not find exact equality. This leads to the idea that by substituting some parameters with actual data, we can determine the theoretical values of the remaining ones. We can substitute historical or implied volatility in this formula in place of σ and correlations between stock prices or between IVs of stocks in place of $corr_{ij}$ to calculate the theoretical risk of an index.

[ii] All charts in this article are provided by Egar Dispersion and IVolatility.com database.

[iii] If the IV of the index was relatively low and lower than HV it would be a good time for buying options on the index, ie to engage in the reverse dispersion strategy.